

Definite Integral.

Riemann Sums.

Remember that:-

$$\sum_{k=1}^n a_k = a_1 + a_2 + \dots + a_n$$

$$\sum_{k=1}^n a = a \sum_{k=1}^n k^0 = n a$$

$$\begin{aligned} \sum_{k=1}^n (a_k - a_{k-1}) &= a_1 - a_0 = a_n - a_0 \\ &+ a_2 - a_1 \\ &+ a_3 - a_2 \\ &+ a_n - a_{n-1} \end{aligned}$$

$$\sum_{k=1}^n k = \frac{n}{2} (n+1) \quad \because \bar{a} > 0$$

$$(\infty + \infty) \frac{\infty}{\infty} = \infty$$

$$\sum_{k=1}^n a^k = \frac{a(1-a^{n+1})}{1-a} \quad \because \bar{a} < 1$$

Some Series:-

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

Bernhard Riemann 1826-1866 :-

$$A = f(\xi_1)(x_1 - x_0) + f(\xi_2)(x_2 - x_1) + \dots + f(\xi_n)(x_n - x_{n-1})$$

$$A = \sum_{k=1}^n f(\xi_k) \Delta x_k$$

$$\Delta x_k = x_k - x_{k-1}$$

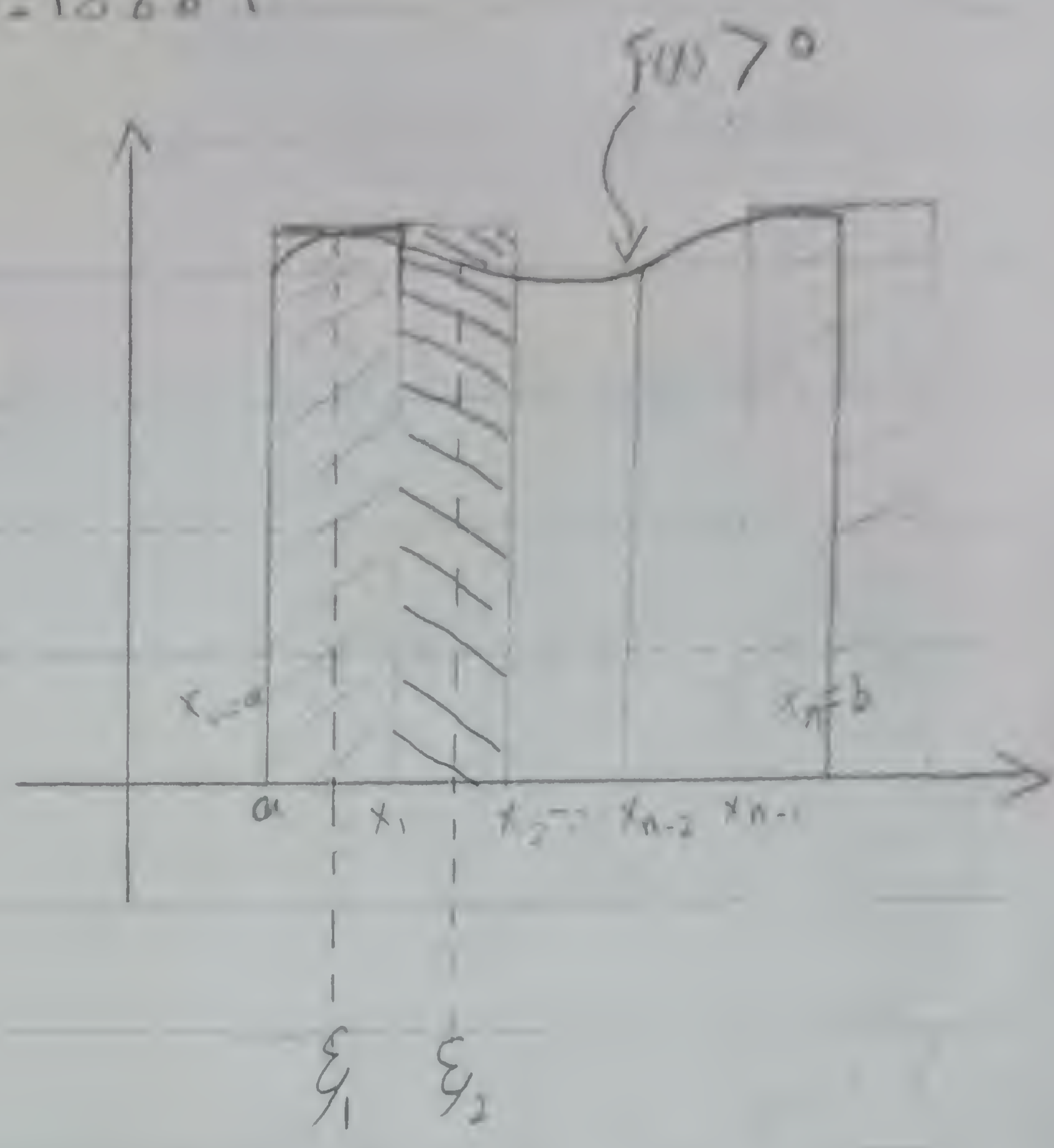
$$\Delta x_k = \frac{b-a}{n}$$

$$x_1 = x_0 + \frac{b-a}{n}$$

$$x_2 = x_1 + \frac{b-a}{n} = x_0 + 2 \frac{b-a}{n}$$

⋮

$$x_n = a + \frac{b-a}{n} k$$



$$x_1 = x_1$$

$$x_2 = x_2$$

$$A = \sum_{k=1}^n F(x_k) \left(\frac{b-a}{n} \right)$$

$$= \sum_{k=1}^n F\left(a + \frac{b-a}{n} k\right) \left(\frac{b-a}{n} \right)$$

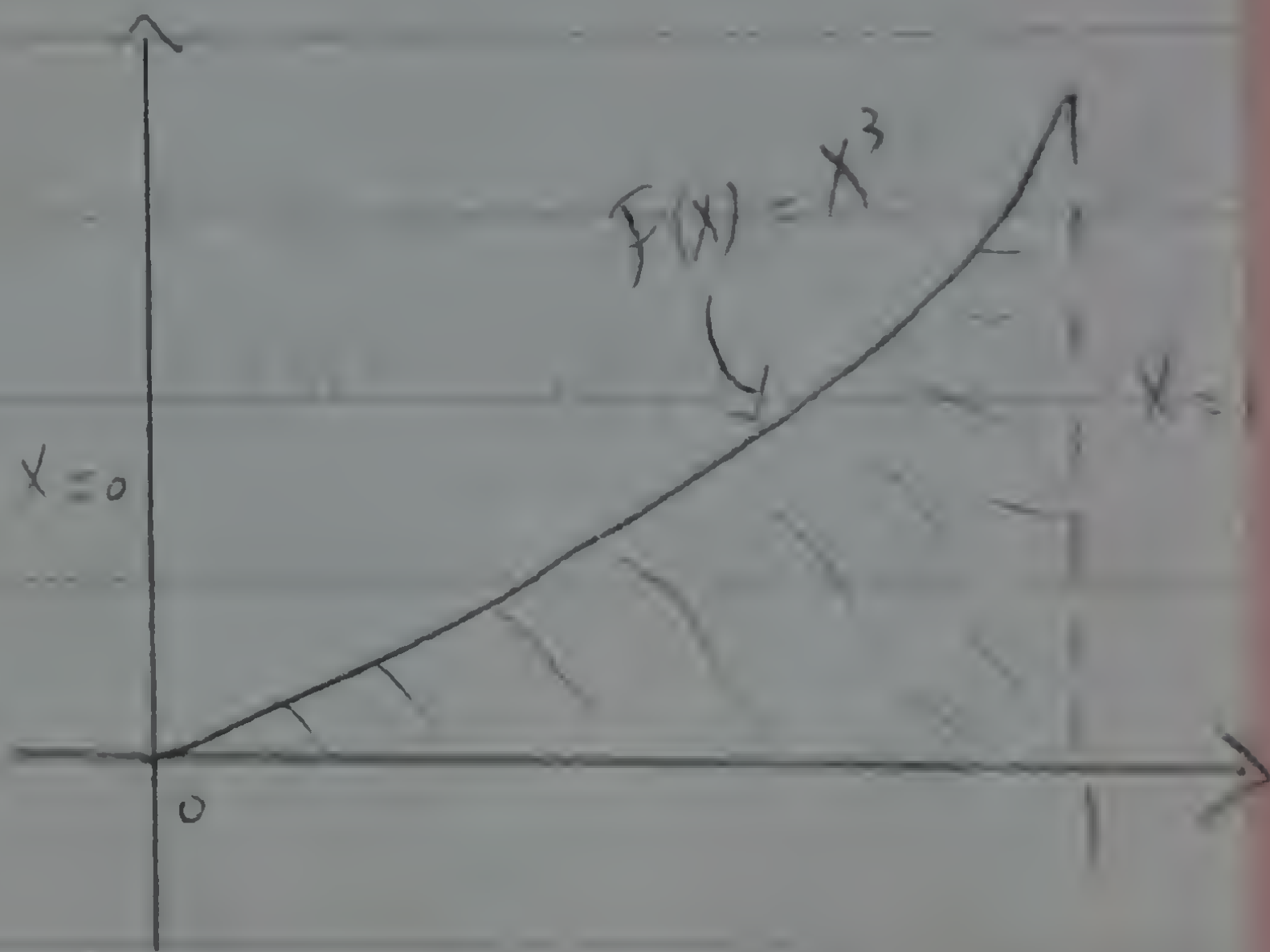
$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{b-a}{n} \right) \cdot F\left(a + \frac{b-a}{n} k\right)$$

EX①: Find the area bounded by $F(x) = x^3$, $x=0$, $y=0$ and $x=1$ using Riemann Sums.

Sol \hookrightarrow

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1-0}{n} \right) \left(0 + \frac{1-0}{n} k \right)^3$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{k^3}{n^3} = \lim_{n \rightarrow \infty} \frac{1}{n^4} \sum_{k=1}^n k^3$$



$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$A = \lim_{n \rightarrow \infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} = \lim_{n \rightarrow \infty} \frac{1}{4} \cdot \frac{n^2}{n^2} \cdot \left(1 + \frac{1}{n}\right)^2 = \left[\frac{1}{4} \right]$$

EX ③: Find the upper and Lower Sums of

$f(x) = x^2$ on the closed interval $[0, 1]$ which correspond the partition:-

$$P_n = \left\{ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \right\}$$

Soln.

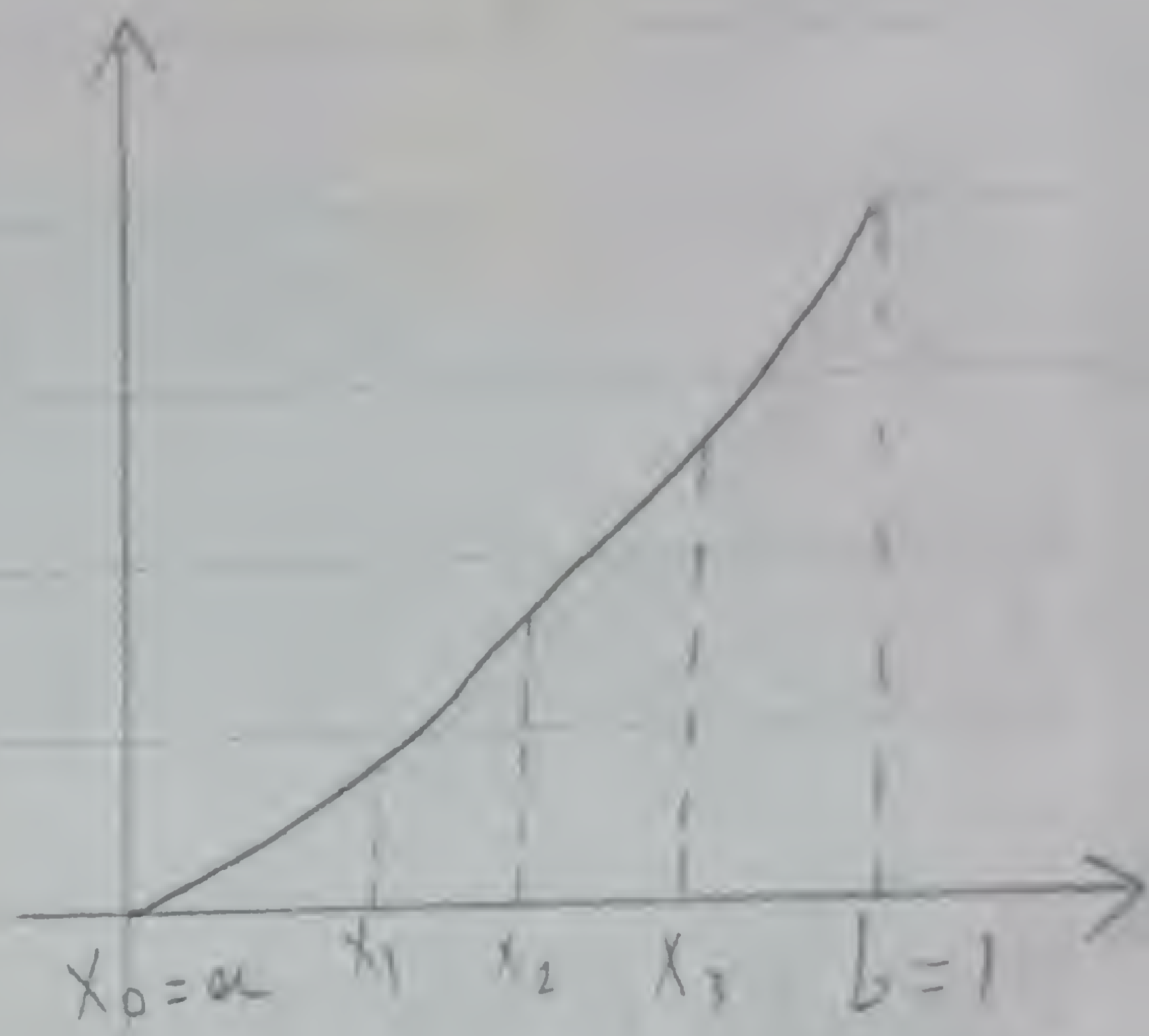
to get Lower Area

$$\text{let } \xi_1 = 0 \quad f(0) = 0$$

$$\xi_2 = \frac{1}{4} \quad f\left(\frac{1}{4}\right) = \frac{1}{16}$$

$$\xi_3 = \frac{1}{2} \quad f\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$\xi_4 = \frac{3}{4} \quad f\left(\frac{3}{4}\right) = \frac{9}{16}$$



$$A_1 = f(0) \frac{1}{4} + f\left(\frac{1}{4}\right) \frac{1}{4} + f\left(\frac{1}{2}\right) \frac{1}{4} + f\left(\frac{3}{4}\right) \frac{1}{4} = \frac{7}{32} \neq$$

to get upper Sums:-

$$\text{let } \xi_1 = \frac{1}{4} \quad f\left(\frac{1}{4}\right) = \frac{1}{16}$$

$$\xi_2 = \frac{1}{2} \quad f\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$\xi_3 = \frac{3}{4} \quad f\left(\frac{3}{4}\right) = \frac{9}{16}$$

$$\xi_4 = 1 \quad f(1) = 1$$

$$A_2 = \frac{1}{16} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4} + \frac{9}{16} \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} = \frac{15}{32} \neq$$

Subject. _____

Date. _____

To you:- Evaluate $\int_1^3 3x^2 dx$
by using Riemann Sums.